

## Ratio of Specific Heats

This investigation considers the ratio of molar specific heats  $\gamma = C_p/C_v$  and various thermodynamic processes involved in its experimental determination.

1. Explain why it is generally true that  $C_p > C_v$ .

2. Consider a diatomic ideal gas.

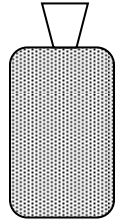
a) Using the equipartition theorem determine  $\gamma$  in 2-dimensions, and 3-dimensions. Explain how you arrived at the appropriate number of degrees of freedom in each case.

b) How would your answers to part (a) change at very low temperatures, and at very high temperatures?

c) Using the first law and gas equation derive the result  $pV^\gamma = \text{const.}$  for an adiabatic process.

3. Consider a gas at pressure  $P_0$  inside a container of volume  $V$ . The container permits the exchange of heat with the environment at a rather slow rate.

a) Describe how  $\gamma$  can be determined by opening the bottle to the atmosphere and then quickly re-plugging it. Assume you have access to a pressure-reading device.



b) Suppose the bottle is unplugged and then quickly re-plugged. Focus attention on the portion of gas that is still within the container after it has been unplugged. Denote atmospheric pressure by  $P_A$  and the final pressure of the gas, long after it has been re-plugged, by  $P_1$ . Draw a  $PV$ -diagram for the processes this portion of gas goes through. Make two separate plots corresponding to  $P_0 > P_A$  and  $P_0 < P_A$ .

c) Determine a relationship between  $\gamma$ ,  $P_0$ ,  $P_A$ , and  $P_1$ .

4. Obtain a bottle, pressure gauge and pump from the front of the room.

a) Pump air into the bottle so that  $P_0 > P_A$ . Unplug and re-plug the bottle and record the final pressure after equilibrium has been reached. Repeat this procedure for three trials. Using the results of the previous section compute your average  $\gamma$ .

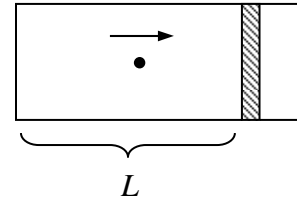
b) Now pump air *out of* the bottle so that  $P_0 < P_A$ . Again unplug and re-plug the bottle and record the final pressure. Repeat for three trials and compute your average  $\gamma$ . Compare with part (a).

c) How does your  $\gamma$  compare with that expected for air? What is  $\gamma$  for a polyatomic gas? What effect would the presence of carbon dioxide or water vapor have on the expected value? Does this improve agreement with your findings?

d) Does the expansion truly correspond to a reversible adiabatic expansion? What effect would irreversibility of the expansion have on your result for  $\gamma$ ?

5. Consider a particle of mass  $m$  bouncing back and forth within a 1-dimensional container of length  $L$ . The container is thermally insulated and one of its ends is a movable piston.

a) Suppose in the figure that the piston moves to the right with speed  $V_0$  and the particle has speed  $v$ . Determine the speed of the particle just after it collides with the piston.



b) If  $V_0 \ll v$  show that the change in the particle's kinetic energy after each collision is:  $\Delta K = -2mvV_0$ .

c) Suppose that the piston moves an amount  $dL$ . This takes an amount of time  $dL/V_0$  over which the particle will have collided many times with the piston. Show that the net change in kinetic energy is:  $dK = -2K dL/L$ . Integrate this result and from your result argue that  $TV^x = \text{const.}$  for a gas of such particles. Determine the exponent  $x$ .

d) The above represents a microscopic consideration of a 1-dimensional adiabatic expansion of a monatomic gas. Using the equipartition theorem determine  $\gamma$  in this instance and show your results are consistent with our previous result  $pV^\gamma = \text{const.}$