

# EXPERIMENT I

## THERMAL EXPANSION

Hollow metal rods of known length and diameter are heated from room temperature to 100°C by passing steam through the rods. The subsequent change in length and diameter is measured. These data allow the determination of the fractional change that occurs in the length per Celsius degree, that is, the coefficient of linear expansion.

### THEORY

When the temperature of a material is changed, the size of an object made from the material also changes. This is caused by a change in the mean interatomic spacing due to a change in the amplitude of vibration of the atoms. In one dimension, the change in length,  $\Delta L$ , of an object is found to be approximately proportional to the original length,  $L_0$ , and also to the temperature change,  $\Delta T$ , where  $\Delta T = T - T_0$ . The proportionality can be expressed in equation form as

$$\Delta L = \alpha L_0 \Delta T, \quad (1)$$

where  $\alpha$  is a constant known as the coefficient of linear expansion of the material. If (1) is solved for  $\alpha$ , then

$$\alpha = \frac{\Delta L}{L_0 \Delta T}, \quad (2)$$

which represents the fractional change in length of the material per Celsius degree.

This analysis can be extended to a two-dimensional object. Figure 1 shows a rectangular plate of length  $L_0$  and width  $W_0$ , at temperature  $T_0$ . The plate can be considered to be made up of thin horizontal rods of length  $L_0$  and of thin vertical rods of length  $W_0$ . When the temperature of the plate changes by  $\Delta T (= T - T_0)$ , the length of the horizontal rods changes by  $\Delta L$ , and the vertical rods change by  $\Delta W$ . Thus the change in the area of the plate,  $\Delta A$ , is found by subtracting the area at temperature  $T_0$  from the area at temperature  $T$ .

$$\Delta A = (L_0 + \Delta L)(W_0 + \Delta W) - L_0 W_0, \quad (3)$$

which can be simplified to

$$\Delta A = 2\alpha A_0 \Delta T, \quad (4)$$

where  $A_0 = L_0 W_0$ , and the small term is neglected. The coefficient of linear expansion is then

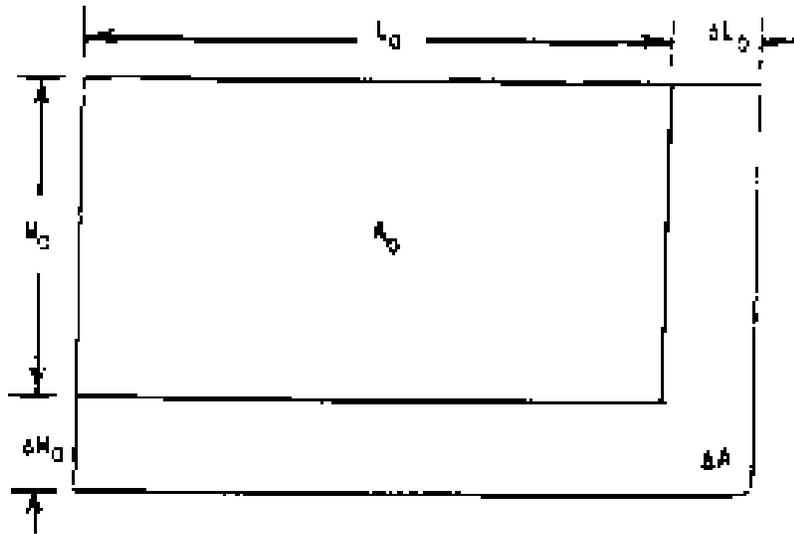


Figure 1. A two-dimensional object whose area has changed by  $\Delta A$  as a result of a temperature change  $\Delta T$ .

The above two-dimensional analysis assumes the material to be Isotropic. In other words, the thermal expansion is uniform in the horizontal and vertical directions. If the material is anisotropic, then two coefficients of linear expansion must be introduced. (See Problem 2.)

If the above analysis is extended to a three-dimensional isotropic material, then an expression similar to (4) is obtained for the increase in volume,  $\Delta V$ ; that is,

$$\Delta V = \beta V_0 \Delta T, \quad (6)$$

where  $\beta$  is the volume coefficient of expansion. For solid isotropic materials,  $\beta=3\alpha$ .

The actual changes in size that occur in objects as a result of a moderate temperature change is very small. For example,  $\alpha$  for aluminum is  $24.0 \times 10^{-6} \text{ (C}^\circ\text{)}^{-1}$ . and the increase in length of a one meter long aluminum rod due to a  $100 \text{ C}^\circ$  increase in temperature is only 2.4 mm. In this experiment, the small changes in a one-dimensional rod are amplified using a linear expansion apparatus, where the expanding rod rests on an axle whose angular change can be observed.

Figure 2 shows that the increase in length,  $\Delta L$ , that occurs as a result of heating, causes the axle on which the rod rests to rotate through an angle  $\theta$ . The change in length,  $\Delta L$ , is the same as the arc length subtended by the angle  $\theta$ . Therefore,  $\Delta L = r\theta$ , where  $r$  is the radius of the axle and  $\theta$  is angular displacement of the rod in radians. When this is substituted into (2), the coefficient of linear expansion can be written as

$$\alpha = \frac{r\theta}{L_0\Delta T} \quad (7)$$

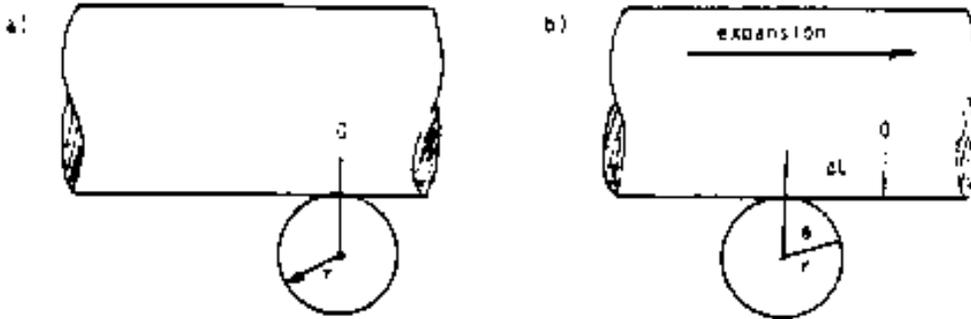


Figure 2. (a) Position of rod on axle prior to heating.  
(b) Position of rod on axle after heating.

An alternative expression for (7) using the measured quantities is

$$\alpha = \frac{\pi}{360Lo\Delta T} \quad (8)$$

where  $\alpha$  is the diameter of the axle and  $\phi$  is the angular displacement of the axle in degrees.

### APPARATUS

- o boiler with rubber tubing and stopper
- o aluminum, brass, and steel rods
- o micrometer
- o knife edge support
- o linear expansion apparatus
- o meter stick
- o small cup
- o electric hot plate
- o 0 -100°C thermometer,  $\pm 2^\circ\text{C}$

Heat should not be applied to the boiler until the apparatus is completely set up and the linear expansion apparatus is zeroed. It may be necessary to hold the rubber stopper down on the boiler until the expansion has ceased. Do not move the apparatus or table during heating. The apparatus is very sensitive to movement.

### PROCEDURE

- a) Use the micrometer to measure the diameter,  $\alpha$ , of the axle on which the rod rests. Record  $\alpha$  and its uncertainty,  $\delta \alpha$ . (Note: In this experiment the lower case delta,  $\delta$ , is used to indicate the uncertainty in a measurement instead of the upper case delta,  $\Delta$ .)

- b) Set up the apparatus as shown in Figure 3. The rod is connected to the rubber tube, the small groove in the rod is placed on the knife-edge, and the rubber sleeve encircling the rod is placed on the axle of the linear expansion apparatus. Pour water into the boiler until it is approximately 1/4 full, and securely plug the boiler with the rubber stopper.

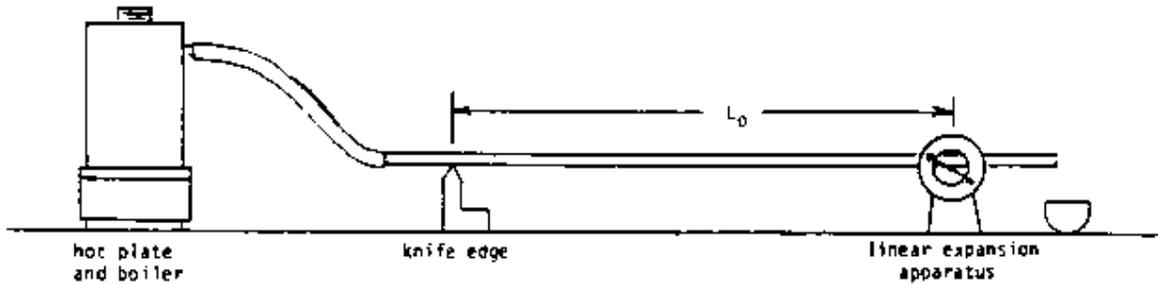


Figure 3. The experimental arrangement.

- c) Use the meter stick to measure the length of the rod from the groove to the position where the rod rests on the axle. This length is  $L_0$ . Also record the uncertainty in length  $\delta L_0$ .
- d) Record the room temperature,  $T_0$ , and the uncertainty in temperature,  $\delta T_0$ .
- e) Zero the linear expansion apparatus. Have the instructor check your apparatus.
- f) Once approval has been obtained, plug in the electric hot plate and heat the water. After a short time steam will emerge from the end of the rod, and the linear expansion apparatus will show an angular displacement.
- g) Wait until the expansion ceases, then record the angular displacement,  $\phi$ , and its uncertainty,  $\delta\phi$ .
- h) Unplug the electric hot plate and allow the rod to cool before removing it.
- i) Repeat the above measurements for the remaining two rods.

## ANALYSIS

Equation (8) is used to calculate the value of the coefficient of linear expansion. The Uncertainty can be found using the "worst case" method. (See Appendix A.) The uncertainty in the coefficient of linear expansion  $\delta\alpha$ , is

$$\delta\alpha = \alpha \left[ \frac{\delta d}{d} + \frac{\delta\phi}{\phi} + \frac{\delta L_0}{L_0} + \frac{\delta(T - T_0)}{T - T_0} \right],$$

Calculate the coefficients of linear expansion and their corresponding uncertainties using (8) and (9). Report these values in a table together with the handbook values. On separate one-dimensional graphs, one for each material, graph the experimental value of  $\alpha$  with its error bars and handbook values. Displace the values vertically from each other so that they can be easily seen.

## QUESTIONS

1. Derive (4) from (3), and explain why a certain term was neglected. Explain the validity in neglecting that term.
2. Suppose that the two-dimensional object in Figure 1 is anisotropic with coefficients of linear expansion  $a_x$  in the horizontal direction and  $a_y$  in the vertical direction. Derive an expression for the increase in area that occurs when the object is heated through a temperature difference  $\Delta T$ .
3. A flat metal washer (a circular plate with a circular hole cut out of it) is heated. Does the hole get smaller, larger, or remain the same size? Explain.
4. Extend the two-dimensional analysis to an isotropic three-dimensional object, and derive the expression for the change in volume that occurs when the temperature of the object changes by  $\Delta T$ .
5. Why is  $L_0$  the distance from the groove to the point of support rather than the entire length of the rod?
6. Start with (8) and derive (9).
7. Why must the increase in length,  $\Delta L$ , be measured so accurately while the length of the rod,  $L_0$  can be measured with a meter stick?