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Math 31 **Activity # 9**

"Solving Quadratic Equations "

Your Name: ____________________

Recall, a **Quadratic Equation** is an equation that includes a variable with an exponent of 2 (**a squared variable**) and no higher powers are on any variable term.

In Standard Form, a quadratic equation whose variable is x looks like:
$ax^2 + bx + c = 0$ (Notice, in the above equation, all terms are on one side and the other side is set equal to 0)
The "a", "b" and "c" represent any real number. Also, "a" = the coefficient on x^2, "b" = the coefficient on **x**, and "c" = the constant

TASK 1

1. Identify "a", "b" and "c" in the following quadratic equations

$x^2 + 6x + 5 = 0$	$2x^2 - 15 = 7x$	$4y^2 = 64$
a = ? b = ? c = ?	a = ? b = ? c = ?	a = ? b = ? c = ?

   (Equations should be in standard form first, before finding "a", "b", and "c", so re-check above, by putting equations in standard form first!)

2. In #1, above, was "a" ever equal to 0?

3. For an equation to remain quadratic, there is one value that "a" can not equal, what value is that?

4. What happens when "a" equals zero, to make the equation not quadratic anymore?

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TASK 2

1. We already learned, one method for solving quadratic equations.
   Do you remember it! Explain that process below.

2. Now, use this method to solve the following quadratic equations.

   a) $x^2 + 6x + 5 = 0$

   b) $2x^2 - 15 = 7x$

   c) $4y^2 = 64$

   d) $y^2 + 2y - 6 = 0$

3. Does #2d above have a solution? (Why or why not?)

2d) above does have a solution, its just that we can't solve it with this current method

TASK 3

1. Now, using the same method from above

   Solve: $x^2 = 9$

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2. How can you solve this equation to get these same solutions without factoring?

3. Try out your new method of **taking square roots** (on both sides) to solve the following: (First, isolate the squared variable term on a side by itself, then take square roots of both sides!)

   a) $x^2 = 49$

   b) $3y^2 - 300 = 0$

   c) $(x - 2)^2 = 25$

   d) $x^2 + 6x + 5 = 0$

4. Why could we NOT solve 3d above by taking square roots?

5. Can we even solve this equation at all? (Hint: See Task 2 #1)

TASK 4

1. So far, we have learned two methods for solving quadratic equations, what are these methods?

   a)

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b)

(We have also seen that these method can NOT be used to solve ALL quadratic equations)

2. Consider: $x^2 + 6x + 9$

a) Factor it:

b) Write the factoring in "exponential form"

"$x^2 + 6x + 9$" is called a **Perfect Square Trinomial** or a **Perfect Square Polynomial** because it comes from *squaring* another polynomial. Perfect Square Trinomials also factor as the square of a polynomial.

2. Identify the "perfect square polynomial" below:
(Hint: Factor the polynomials to help identify which one is a perfect square polynomial)

a) $y^2 - 8y + 16$

b) $x^2 + 5x + 4$

Which one is the perfect square polynomial?

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3. In #s 1 & 2 above, we have two perfect square polynomials. Fill in the chart below.

Perfect Square Polynomial	Constant	Middle Coefficient
$x^2 + 6x + 9$		
$y^2 - 8y + 16$		

4. For each perfect square polynomial, compare the constant with the middle coefficient and try to create a formula that expresses the constant in terms of the middle coefficient. (This relationship should work for both polynomials)

   a) Express 9 in terms of 6

   b) Express 16 in terms of -8

5. Let's verify this relationship in the following perfect square polynomials.

   a) $x^2 + 10x + 25$

   b) $y^2 - 18y + 81$

We can thus create our own Perfect Square Polynomials by adding in the constant that gives the relationship with the middle coefficient. This process is called **"Completing The Square."**

6. Add the constant that completes the square in the following:

   a) $x^2 + 14x + ?$

   b) $y^2 - 12y + ?$

   c) $x^2 + 3x + ?$

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7. Now, rewrite the above polynomials with the constants included, then factor these polynomials, and put the factoring in exponential form when possible.

a) b) c)

8. How do the numbers used in your factorings in #7, relate to the middle coefficients in #6?

To complete the square on:	Add $(\frac{1}{2}\cdot b)^2$, to get:	This polynomial will now factor as:
$x^2 + bx$	$x^2 + bx + \mathbf{(\frac{1}{2}\cdot b)^2}$	$(x + \mathbf{\frac{1}{2}\cdot b)^2}$

9. Use the chart above to complete the square and factor the following:

$x^2 + 16x$		
$x^2 - 20x$		
$x^2 + 7x$		

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We can use "Completing the Square" to help set up Quadratic Equations so we can further solve by "Taking Square Roots."

Recall, when we tried to solve $x^2 + 6x + 5 = 0$ (Task 3, #3d), we couldn't solve by taking square roots. We now can set this equation up so that we will be able to eventually take the square roots of both sides and solve for the variable.

10. Let's use the chart below to solve: $x^2 + 6x + 5 = 0$ by completing the square.

Solve: $x^2 + 6x + 5 = 0$	Calculations
1st) Move the constant to the other side	
2nd) Complete the square (Recall, what you do to one side of an equation, you must do to the other side)	
3rd) Factor the side with the polynomial	
4th) Now, take square root of both sides	
5th) Further solve for x	
6th) Is this the same solution you arrived at in Task 3, #3d?	

TASK 5

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1. What was the coefficient on x^2 (the quadratic term or squared variable term) in the examples in Task 4?

To solve Quadratic Equations using Completing the Square, the **coefficient on the quadratic term must be 1.**

2. Consider the equation: $4x^2 - 12x + 24 = 0$

   How can we make the coefficient on x^2 a 1, when it is not a 1 at first?

3. Solve: $3x^2 - 15x + 18 = 0$ (Repeat the steps from TASK 4, #10)

Solve: $3x^2 - 15x + 18 = 0$	Calculations
Get Coefficient of 1 on quadratic term	
1st) Move the constant to the other side	
2nd) Complete the square (Recall, what you do to one side of an equation, you must do to the other side)	
3rd) Factor the side with the polynomial	
4th) Now, take square root of both sides	
5th) Further solve for x	

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6th) Check your solution(s)	

TASK 6
There is still one more method for solving quadratic equations.
This method is somewhat simple because it is a formula that simply leads to the solution.
All, we need to do is make substitutions in the formula then simplify to find the value(s) for the solution(s).

This formula is called **"The Quadratic Formula"**

To solve for "x" in the equation: $ax^2 + bx + c = 0$,

The Quadratic Formula is: $X = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

This formula is the answer, we just substitute the values for "a", "b" and "c" from the equation into this formula, and further simplify

1. Let's use the quadratic equation to solve some of the previous equations. Since we already solved these equations before, we should get the same answers as we got before.

   First identify what: a = ? b = ? c = ?

   a) $x^2 + 5x + 6 = 0$

   b) $(x - 2)^2 = 25$

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c) $3x^2 - 15x + 18 = 0$

d) $y^2 + 7y = 9$

e) $y^2 + 2y - 6 = 0$